(Date)
(P No)

Assignment = 05

Q1 State Stokes Theorem for Convergence.

Ans If C is a simple closed curve which encloses a surface S and $\vec{F}(x,y,z)$ is a vector function which is continuous at every point of S. then.

$$\oint_C \vec{F}\cdot d\vec{r} = \int_S (\nabla \times \vec{F})\cdot \hat{n}\, \partial s$$

Q2 find the Curl of the vector field $\vec{P} = x^2\hat{i} + y\hat{j} + 2z\hat{k}$ at the point (1, 2, 2).

Ans $\text{curl}.\vec{F} = \nabla\times\vec{F} = \left(\frac{\partial}{\partial y}f_3 - \frac{\partial}{\partial z}f_2\right)\hat{i}$

$$-\left(\frac{\partial}{\partial x}f_3 - \frac{\partial}{\partial z}f_1\right)\hat{j}$$

$$+\left(\frac{\partial}{\partial x}f_2 - \frac{\partial}{\partial y}f_1\right)\hat{k}$$

$\Rightarrow \vec{P} = \vec{F} = x^2\hat{i} + y\hat{j} + 2z\hat{k}$

$\Rightarrow f_1 = x^2, \quad f_2 = y, \quad f_3 = 2z$

Now,

$$\nabla\times\vec{F} = \left(\frac{\partial(2z)}{\partial y} - \frac{\partial}{\partial z}y\right)\hat{i} - \left(\frac{\partial(2z)}{\partial x} - \frac{\partial}{\partial z}x^2\right)\hat{j}$$

$$+\left(\frac{\partial}{\partial x}y - \frac{\partial}{\partial y}x^2\right)\hat{k}$$

Date
P No.

$$\boxed{\nabla \times \vec{F} = 0}$$ Ans

Q3 Find the Directional Derivative of $f(x,y,z) = x + yz^3$ in the Direction of the Vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ at the point $(0,-1,1)$

As Given,

$$\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$
$$f(x,y,z) = x + yz^3$$

Now,
$$|\vec{u}| = \sqrt{4+9+16} = \sqrt{29}$$

$$\nabla . \vec{f} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$= \hat{i}\frac{\partial (x+yz^3)}{\partial x} + \hat{j}\frac{\partial (x+yz^3)}{\partial y} + \hat{k}\frac{\partial (x+yz^3)}{\partial z}$$

$$\Rightarrow 1\hat{i} + z^3\hat{j} + 3yz^2\hat{k}$$

$$(\nabla . f)_{(0,-1,1)} = 1\hat{i} + \hat{j} + 3(-1)(1)^2\hat{k}$$
$$= \hat{i} + \hat{j} - 3\hat{k}$$

Now,
$$D.D = \nabla f|_p . \hat{u} = \frac{(\hat{i} + \hat{j} - 3\hat{k}).(2\hat{i} + 3\hat{j} + 4\hat{k})}{\sqrt{29}}$$

Date
P No

$$D.D = \frac{2+3-12}{\sqrt{29}} = \frac{-7}{\sqrt{29}}$$ Ans

Q4 If V is the volume enclosed by a closed surface S and $\vec{F} = 2x\hat{i} + y\hat{j} + z\hat{k}$. find $\int \vec{F}.\hat{n}\,ds$

Sol Given that,

$$\vec{F} = 2x\hat{i} + y\hat{j} + z\hat{k}$$

To find:

$$\int \vec{F}.\hat{n}\,ds = \iiint_V \nabla.\vec{F}\,dv$$

Now,

$$\nabla.\vec{F} = \frac{\partial(2x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial z} = 2+1+1 = 4$$

So,

Flux through the surface S enclosing the volume V is ;

$$\boxed{\iiint_V \nabla.\vec{F}\,dv = \phi = 4V}$$

Ans

Date:
P. No:

Q5 If f is a homogeneous function of degree n, then Justify

$$x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = n(n-1)f$$

Sol

Given that,

f is a homogenous of degree n.

Now,

By Euler's Theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf \quad -(1)$$

Partially differentiate (1) w.r.t. x and w.r.t. y seperately.

$$x\frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial x} + y\frac{\partial^2 f}{\partial x \partial y} = n\frac{\partial f}{\partial x} \quad -(2)$$

$$x\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial y^2} = n\frac{\partial f}{\partial y} \quad -(3)$$

(2) * x and (3) * y

Then Add Both eq.

Date
P No

$$x^2\frac{\partial^2 f}{\partial x^2} + x\frac{\partial f}{\partial x} + xy\frac{\partial^2 f}{\partial x \partial y} = nx\frac{\partial f}{\partial x} \quad \text{— (A)}$$

$$xy\frac{\partial^2 f}{\partial x \partial y} + y\frac{\partial f}{\partial y} + y^2\frac{\partial^2 f}{\partial y^2} = ny\frac{\partial f}{\partial y} \quad \text{— (B)}$$

Now

(A) + (B)

$$\Rightarrow x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = (n-1)x\frac{\partial f}{\partial x} + (n-1)y\frac{\partial f}{\partial y}$$

$$\Rightarrow x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = (n-1)\left[x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}\right]$$

$$\left[x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf\right]$$

$$\Rightarrow x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = n(n-1)f$$

Hence Prove.